A COMPUTATIONAL SUBSONIC AEROELASTIC ANALYSIS OF A THIN FLAT PLATE

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Abstract The work described in this paper is part of a computational parametric aeroelastic analysis of a thin flat plate clamped at the leading edge and exposed to subsonic airflow. The plate's equation of motion was modeled using Newton's Law of Motion while the airflow was modeled using the Small Disturbance Unsteady Aerodynamic equation. The aeroelastic analyses were performed at various aspect and mass ratio and were validated with published work. Results had shown that the flutter velocity and flutter frequency decreases as the aspect ratio and mass ratio of the plate increases.

Keywords: Subsonic flow, Flutter, Aspect ratio, Mass ratio.

INTRODUCTION

Modern aircraft structures are extremely flexible and therefore tend to deform when exposed to airflow [1]. This usually involves interaction of the inertial, elastic and aerodynamic forces, which can severely affect the stability, performance and manoeuvrability of the aircraft. Because of the practical consequences of aeroelasticity, understanding of the aeroelastic behaviour is critical. And with the rapid development in computational technology, many research works in aeroelastic studies are focussed on developing efficient and robust computational tools that can model all the important characteristics of the interaction.

This study, therefore, is an attempt to look into the aeroelastic behaviour of a thin flat plate clamped at the leading edge under subsonic flow. It is predicted that the plate will undergo 'flutter'. Flutter is a type of self-excited vibration that occurs above a certain 'critical flutter speed' in which the initial perturbation is provided by the air stream [2]. The mode of failure is often sudden and destructive in nature. It is commonly encountered on lifting surfaces in supersonic flow and often modeled as panels or plates with fixed or pinned supports on all four sides with one side of the surface exposed to an air stream [2].

Previous works on subsonic panel aeroelasticity had focussed on determining the mode of instability. Kornecki [3] theoretically studied the aeroelastic behaviour of two-dimensional flat panels clamped at both edges and found that the panel lost its stability due to divergence (buckling) in subsonic flow and flutter in supersonic flow. Kornecki et al. [4] extended the work to cantilevered panels, which were shown to flutter in subsonic flow. Shayo [5] later extended the work to three-dimensional with the inclusion of downstream wake effects and concluded that the wake effects can be significant when the mass ratio is large. Subsequently, there was very few works conducted on subsonic panel flutter particularly on the parametric analysis of the flutter characteristics.

This paper presents the results of a parametric computational study on the flutter characteristics of a thin flat plate. The parameters considered are the aspect ratio, mass ratio and number of structural modes in the structural model. The results of the study were verified with the experimental and analytical results obtained by Kornecki et al. [4].

THEORY AND MODELLING

Assuming zero damping, the governing equation of motion of an aeroelastic system in matrix form [6] is given as

$$\left[\overline{\mathrm{M}}\right]\left\{\ddot{\chi}(\mathrm{t})\right\}+\left[\overline{\mathrm{K}}\right]\left\{\chi(\mathrm{t})\right\}-\left\{\mathrm{F}_{\mathrm{a}}(\chi)\right\}=0 \qquad (1)$$

where $\left[\overline{\mathrm{M}}\right]$ and $\left[\overline{\mathrm{K}}\right]$ are the mass and stiffness matrices, $\{\chi(t)\}$ is the structural deformation and $\{F_a(\chi)\}$ is the aerodynamic forces induced by the structural deformation. $\{F_a(\chi)\}$ is computed based upon the linearised unsteady, small disturbance equation [6].

To solve the aeroelastic equation of motion, the modal approach [6] is used which can be expressed as

$$\{\chi\}=[\varphi]\{\overline{\mathrm{q}}\} \qquad (2)$$

where [φ] is the modal matrix whose columns contain the lower order of natural modes and $\{\overline{q}\}$ is the generalized coordinates. In this approach, the structural deformation of the flutter mode can be sufficiently represented by the superposition of lower order structural modes contained in [φ]. These lower order structural modes can be generated through a separate eigenvalue analysis of the undamped free vibration equation. Therefore, Eq. (1) can be expressed in terms of generalized coordinates given as [6]

$$\left[-\omega^2 M + K - q_\infty Q(ik)\right]\{\bar{q}\} = 0 \quad (3)$$

Eq. (3) represents the flutter equation of motion whereby *Q(ik)* is the generalized aerodynamic forces, ω and $\{\bar{q}\}$ are the unknown natural frequencies and mode shapes of the flutter modes. Eq. (3) is solved in the reduced frequency, *k*, domain and finally the flutter velocity, U_f and flutter frequency, ω_f, are obtained from the following equation [6]

$$k = \frac{\omega_f L}{U_f} \quad (4)$$

COMPUTATIONAL PROCEDURE

Using a commercial finite element code, MSC-NASTRAN, the structural modes of a flat plate were computed. The plate geometry was meshed into 40 elements and the modes were obtained using MSC-NASTRAN's eigenvalue analysis, which solves the generalized eigenvalue problem of

$$[K]\{\hat{\chi}\} = \lambda[M]\{\hat{\chi}\} \quad (5)$$

where λ and $\hat{\chi}$ are the eigenvalue and eigenvector associated with the natural frequency and mode shape of the plate. The free vibration modes were then used for the analysis of flutter in ZAERO.

Unlike NASTRAN, ZAERO utilises the panel method to calculate the aerodynamic loading and obtain aeroelastic solutions. Therefore, the plate geometry is discretized into 40 panels as shown in figure 1. For subsonic Mach number, the location of the control point in the aerodynamic panel is at mid-span and 85% of chord length of the box [6].

Upon meshing of the plate, vortex singularity distributions of unknown strength are approximated over the surface of these panels which were later transformed into aerodynamic influence coefficient matrix, [AIC]. This matrix relates the structural deformation to the aerodynamic forces.

The unsteady pressure coefficient distribution due to the plate's motion can then be determined by imposing the boundary conditions (structural modes) on the aerodynamic panels. Once the pressure distribution is known, the generalized aerodynamic forces can be expressed in terms of the mode functions, *h* and the pressure coefficient, C_p [6]

$$Q_{IJ} = \sum_{0}^{40} C_{p_i}^{(J)} A_i h_i^{(I)} \quad \textbf{(6)}$$

where $C_{p_i}^{(J)}$ is the pressure coefficient of the i^{th} panel due to the J^{th} mode and A_i is the panel area. Then, the flutter equation (see Eq. 3) can be solved using g- and K-methods [6], which are provided in ZAERO. The g-method provides the flutter solutions as a function of velocity whereas in K-method (also known as V-g or American method), the solutions are given as a function of reduced frequency. In ZAERO, the flutter boundary is obtained when the aerodynamic damping, *g* is zero which indicates a self-excited vibration.

However, the problem of transferring data between the panel model and the finite element model arises as a result of the different discretization techniques employed in both numerical methods. Therefore, an interpolation matrix required for the transformation is used. In this study, the Infinite-Plate-Spline (IPS) technique, based on the partial differential equation of equilibrium for an infinite plate, is employed as it is most suited for plate (wing-like components). The spline matrix generated, *[G]*, relates the interpolated displacement vector, *{h}* at aerodynamic control points with the structural displacement, $\{\chi\}$ at structural nodes [6]:

$$\{h\} = [G]\{\chi\} \quad (7)$$

Based on the principle of virtual work, the aerodynamic force transferred from the aerodynamic model to structural model may be written as [6]:

$$\{F_a\} = [G]^T\{F_h\} \quad (8)$$

The transformation strategy can be described as follows:

1. The displacements, χ, obtained from NASTRAN's output file are projected on the aerodynamic grid points.
2. Calculate the interpolated displacements, *h*, at the aerodynamic control points.
3. Obtain the aerodynamic forces, F_h, due to *h*, at aerodynamic control points.
4. Interpolate F_h to aerodynamic forces, F_a at aerodynamic grid points.
5. Calculate the new displacements, χ, (using g- and K-method) at aerodynamic grid points due to F_a.
6. The flutter mode shape is given by the new displacements.

RESULTS AND DISCUSSION

Validation of Flutter

The flutter analyses were performed for a square panel with the properties as shown in Table 1. The flutter frequency and velocity for these two methods are then compared with published results [4] and are given in the following Table 2.

Table 1: Material properties of Aluminum plate

Modulus of Elasticity, E	6.8947e+10 Pa
Poisson's Ratio, ν	0.3
Mass density, ρ_m	2643.38 kg/m^3
Mass Ratio	0.232
Thickness, h_m	0.000508 m

Table 2: Comparison of flutter frequency and velocity

Method	Flutter freq. (Hz)	Flutter Velocity (m/s)
Experiment [4]	40.2	27.4
Non-circ. Theory [4]	37.5	24.0
Quasi-steady theory [4]	42.6	1.8
Full unsteady theory [4]	39.8	18.3
ZAERO g-method [7]	35.8	31.5
ZAERO K-method [7]	35.7	31.5

The results using the non-circulatory flow theory are in excellent agreement with experiment, however it neglects the circulatory effect, which becomes significant in high frequency oscillation. The quasi-steady theory is also inadequate since it neglects the effect of wake vortices, predicting a very low flutter frequency. The flutter frequency calculated using the full unsteady theory, however, showed excellent agreement with experimental but predicted a low flutter velocity. In comparison, the flutter frequency and velocity obtained from both ZAERO's g- and K-methods showed fair agreement with the experimental results.

The analysis was extended to mass ratios of 0.1, 0.2, 0.3, 0.4 and 0.5 and the corresponding flutter results were obtained. The non-dimensional flutter frequency and dynamic pressure from both g- and K-methods are then compared with results from the same reference. It was found that the results using ZAERO were comparable with experimental and theoretical results from reference [4] as shown in figures 2(a) and 2(b).

Effect of Aspect Ratio and Mass Ratio

Aspect ratio is defined as the ratio of the span length over the chord length of the plate. The inclusion of higher structural modes in the flutter analysis is important with increases in the aspect ratio of the plate. However, employing a large number of modes requires finer discretization of the finite element model in which the present study is limited to a size of 40 elements. Therefore, the parametric analyses were performed using only 6 modes, which potentially may give an acceptable percentage error of only 2%. Figure 3 showed a representation of the structural mode shapes from mode 1 to mode 6 for a plate with an aspect ratio of 1 whereas figure 4 presented the flutter mode shapes at different time interval for the same plate. The flutter mode shapes assumed the mode shape of structural mode 2 as shown in figure 3.

Dowell [8] stated that the effect of aspect ratio is significant when it is higher than 1. Plots of the aerodynamic damping versus the freestream velocity for aspect ratio of 5 to 20 are shown in figure 5. The zero damping crossings were shown to occur at lower flutter velocities as the aspect ratio increases. The damping curve of aspect ratio of 5 is of the divergent type, which appeared to change to the hump mode type at larger aspect ratio. The small positive damping values associated with this type of mode indicate points of high instability [9].

Similar plots for varying mass ratios of 0.1 to 0.5 are shown in figure 6. Again, the zero damping crossings occurred at lower flutter velocities as the mass ratio increases. All the damping curves are of the divergent type. The slopes of the curves appeared steeper with increasing mass ratio, indicating a more violent flutter behaviour [9].

Figures 7 and 8 presented the flutter velocities and flutter frequencies versus the mass ratio for various aspect ratio panels. Both figures showed that the flutter velocity and flutter frequency decreases as the mass ratio and aspect ratio increases. Flutter is primarily influenced by the stiffness of a structure. At higher aspect ratio and mass ratio, the plate is actually less stiff, which reduces it to behave more like a beam. Hence the plate tends to oscillate at lower velocity and frequency. This in agreement with the observation noted by Kornecki et al. [4] and Shayo [5].

CONCLUSION

A computational aeroelastic analysis of a thin flat plate under subsonic flow is presented. This study clearly shows that the effects of the aspect ratio and mass ratio are significant in characterising the flutter boundary.

NOMENCLATURE

g	aerodynamic damping
k	reduced frequency
l	length of plate
q_∞	freestream dynamic pressure
ρ_a	air density
$[\varphi]$	modal matrix
ω	natural frequency
ω_f	flutter frequency
λ	eigenvalue
A	panel area
C_p	pressure coefficient
D	flexural rigidity
$\{F_a(\chi)\}$	aerodynamic force induced by structural deformation
$\{h\}$	interpolated deformation at aerodynamic boxes
$[K]$	generalized stiffness matrix
$[\overline{K}]$	stiffness matrix
L	reference length of plate, half span
$[M]$	generalized mass matrix
$[\overline{M}]$	mass matrix
$Q(ik)$	generalized aerodynamic force
$\{\overline{q}\}$	generalized coordinates
U_f	flutter velocity
$\{\hat{\chi}\}$	eigenvector
$[AIC]$	aerodynamic influence coefficient matrix
$\dfrac{\rho_a l}{\rho_m h_m}$	mass ratio
$\omega_f\sqrt{\dfrac{\rho_m h_m l^4}{D}}$	non-dimensional frequency
$\sqrt{\dfrac{\rho_m U_f^2 l^3}{D}}$	non-dimensional dynamic pressure

REFERENCE

Megson, T. H. Aircraft Structures For Engineering Students, Hodder & Stoughton, Great Britain, 1990.

Scanlan, R. H and Rosenbaum, R. Introduction to the Study of Aircraft Vibration and Flutter, Dover Publications Inc., NY, 1968.

Kornecki, A. "Static and Dynamic Instability of Panels and Cylindrical Shells in Subsonic Potential Flow", Journal of Sound and Vibration, 32(2), 1974, pp. 251-263.

Kornecki, A., Dowell, E. H. and O'Brien, J. "On the Aeroelastic Instability of Two-Dimensional Panels in Uniform Incompressible Flow", Journal of Sound and Vibration, 47(2), 1976, pp. 163-178.

Shayo, L. K. "The Stability of Cantilever Panels in Uniform Incompressible Flow", Journal of Sound and Vibration, Vol. 68, No. 3, 1980, pp. 341-350.

Zona Technology Inc. ZAERO Theoretical Manual, Ver. 4.1, Jan. 2000.

Majid, D. L. "Subsonic Aeroelastic Analysis of a Thin Flat Plate" Master Thesis, 2001.

Dowell, E. H. Aeroelasticity of Plates and Shells, Noordhoff International Publishing, Leyden, 1975.

Bisplinghoff, R. L. and Ashley, H. Principles of Aeroelasticity, Dover Publications Inc., 1962

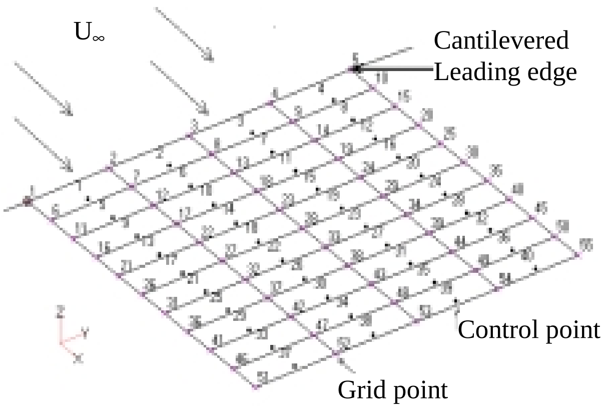
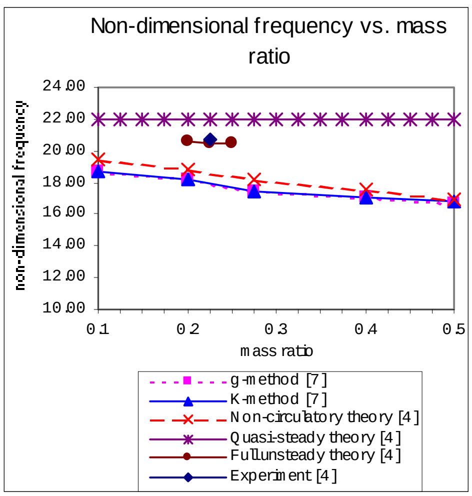
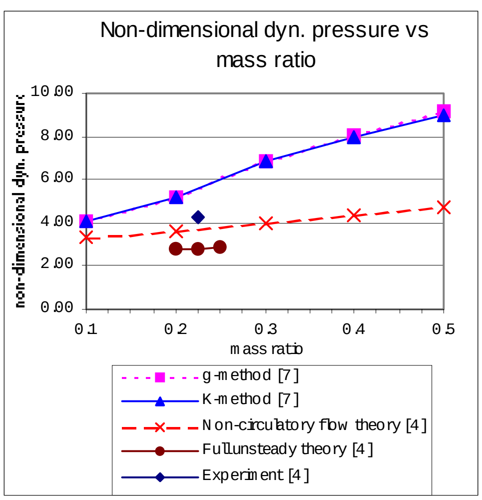


Figure 1: Aerodynamic Model with top and bottom flow



(a)



(b)

Figure 2: Comparison of a) non-dimensional frequency; and b) non-dimensional dynamic pressure.

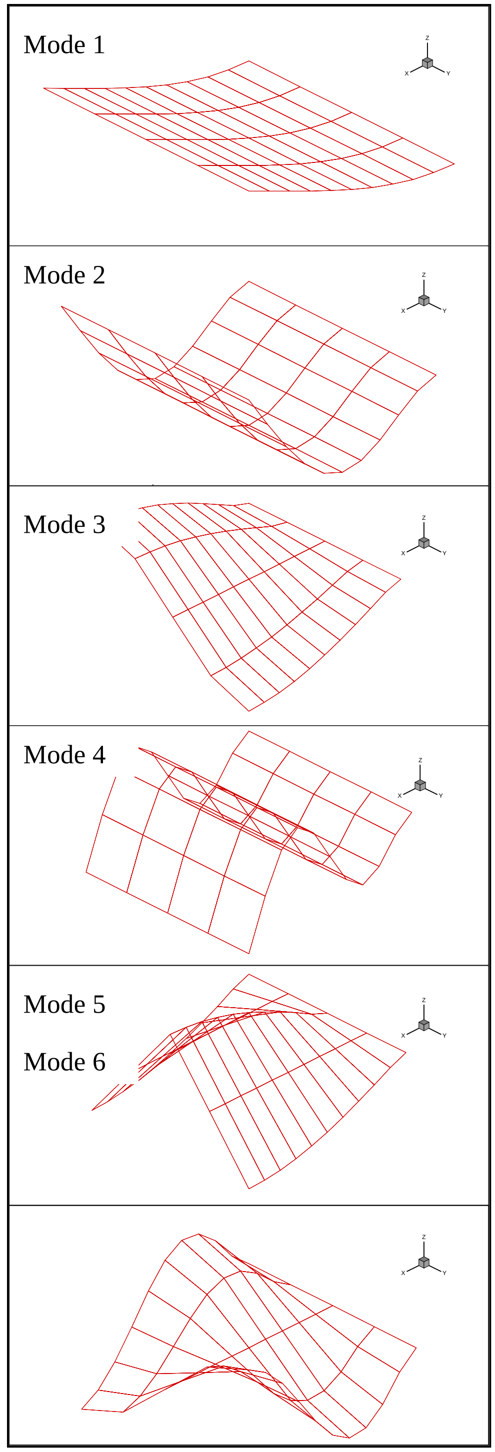


Figure 3: Structural mode shapes from mode 1 to mode 6 for A Plate with Aspect Ratio of 1

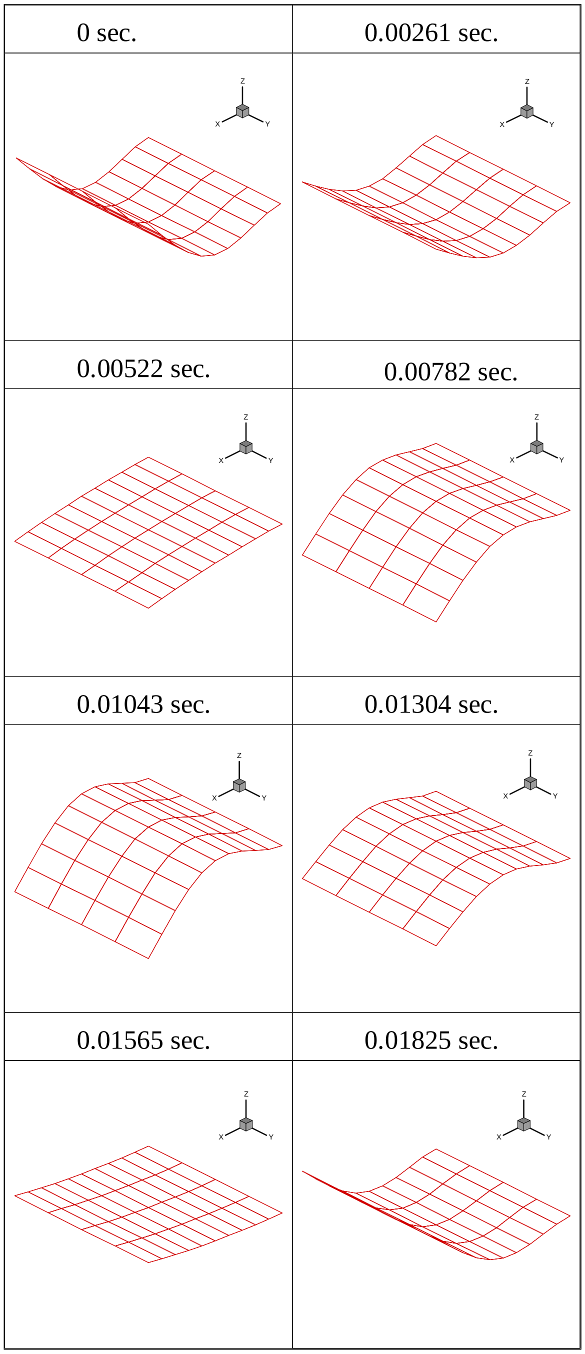


Figure 4: Flutter Mode Shapes At Different Time Interval for A Plate with Aspect Ratio of 1

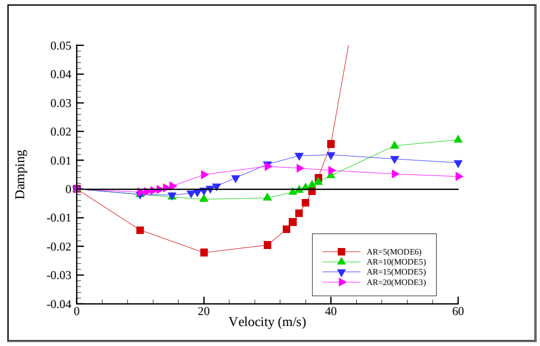


Figure 5: Damping versus Freestream Velocity at Large Aspect Ratio (>4)

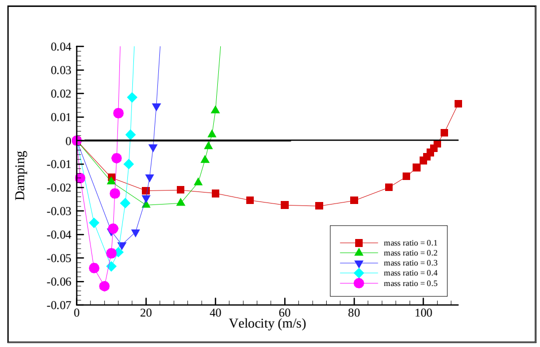


Figure 6: Damping versus Freestream Velocity at Different Mass Ratio

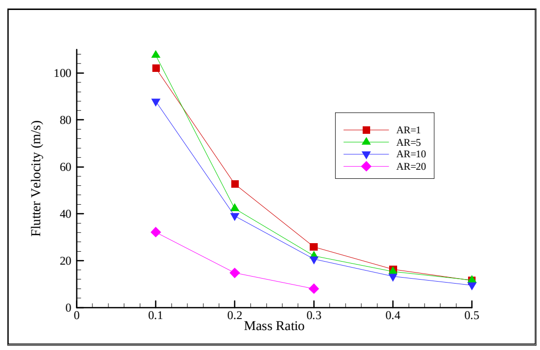


Figure 7: Flutter Velocity versus Mass Ratio at Different Aspect Ratio

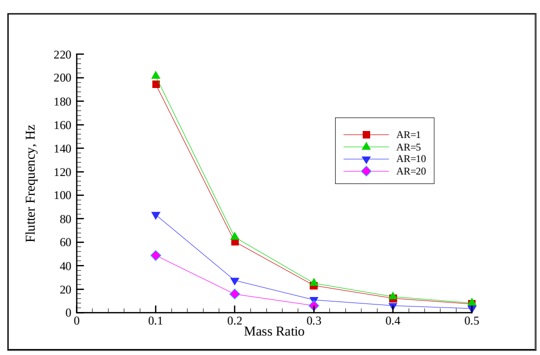


Figure 8: Flutter Frequency versus Mass Ratio at Different Aspect Ratio